Estimating Beech Growth and Survival
A study based on longterm experiments in Slovakia

(With 3 Figures and 5 Tables)

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1. INTRODUCTION

The change from the traditional yield tables to individual tree growth models for beech has been met with enthusiasm as well as scepticism. Yield tables are easy to use, and they are „robust“, which means that predictions are usually found to be sufficiently reliable by practitioners, provided the yield table predictions are calibrated with the measured inventory data. Growth estimates for individual trees on the other hand, have many sources of error and simulated graphics may create a false impression that all the predictions are accurate. Comparisons of the actual growth with the predictions are rare, especially at the extremes of density, age and growing site; the few existing ones give rise to scepticism (WINDHAGER, 1999; GADOW and HEYDECKE, 2000). In addition to the greater uncertainty, individual tree growth models have a practical disadvantage when compared with a stand model. Harvest events can be defined by selecting individual trees for removal on a computer screen. But simulating alternative management regimes for a particular stand requires specification of numerous harvest events which should be more or less consistent with the language that foresters use. Thus, there is a need for simpler models of forest dynamics. Such models should be able to a) simulate harvest events, i.e. estimate removals using existing silvicultural terminology and b) predict the growth response following a particular harvest event.

This paper presents an attempt to achieve these two objectives, based on longterm growth studies of European Beech. The European Beech (*Fagus sylvatica* L.) is the dominant tree species in central Europe. It prefers a maritime climate with moderate fluctuations of temperature and precipitation and is rarely found in areas with extended very cold and dry periods. The European Beech is shade tolerant and prefers moist sites. The geographic range where beech is dominant and the range where it occurs, have been described in numerous research papers[1]. According to BOLTE et al. (2007) more than 20 maps of its phytogeographical range have been published.

Possibly the first approach to beech growth and yield modeling was the yield table published by PAULSEN (1795). These and subsequent generations of yield tables portray the development of pure beech stands in regular time intervals for a fixed silvicultural treatment and site quality. The empirical data base of beech growth observations was considerably improved and extended since the end of the 19th and during the 20th century. The most widely used German yield tables published by SCHOBER (1972) and DITTMAR et al. (1986) are based on the works of SCHWAPPACH (1911) and WIEDEMANN (1931) who had been the first custodians of the extensive database of the Prussian Forest Research Institute.

During the second half of the 20th century, many yield tables were supplemented or even replaced by flexible stand growth models. Due to improved computer technology and increasing information needs, several innovative single tree growth simulators were developed during the past two decades to evaluate alternative silvicultural strategies (HASENAUER, 1994; STERBA and MONSERUD, 1997; PRETZSCH and KAHN, 1998; PRETZSCH et. al., 2002; NAGEL et al., 2002). It is generally assumed that single tree models are especially useful in uneven-aged forests with varying densities and species combinations. However, the accuracy of their projections, which cannot be estimated for the range of potential applications, remains to be uncertain. One of the reasons may be that the empirical database is heavily biased towards normal growing conditions and normal silviculture (WINDHAGER, 1999; GADOW and HEYDECKE, 2000). Thus, the dramatic change from the traditional yield tables to individual tree growth models for beech has been met, not only with enthusiasm, but also with some scepticism. Based on our extensive experience with individual tree models and their uncertainties, we anticipate that it is preferable to first develop a sound stand level model and to use that model to constrain the predictions for individual trees.

The objective of this paper is to complement the range of existing beech growth models with a stand-level approach. Specifically, it should be possible to use the dynamic growth model to simulate alternative silvicultural options for prediction intervals of varying lengths. An important requirement of the model, besides accurate and unbiased growth predictions, is ease of specifying silvicultural treatments. Thus the model will be designed to facilitate silvicultural planning and economic analysis of alternative management options.

2. THE DATASET

The data used in this study originate from the growth and yield database of the Forest Research Institute of the Slovak Republic. The data were collected on 47 long-term beech research plots in 13 locations, most of them at altitudes between 350-800 m in the Carpathian Mountains and the Slovak plains. The distribution of research plot locations is shown in *Fig. 1*. The research plots were established for the purpose of constructing yield tables for Czechoslovakia during the period 1965–1973 (HALAJ et al., 1987, 1988; HALAJ and PETRÁŠ, 1998).

Some of the plots which are concentrated at the same location are thinning experiments where different thinning types were applied. The plots were re-measured between one and 6 times from 1969 to 1995. The shortest observation interval between plot re-measurements is five years, the longest 18 years. The mean, maximum and minimum values and the standard deviation of the main stand variables are shown in *Table 1*. The range of ages, and stand densities and thinning intensities is wide and covers a great variety of conditions and silvicultural treatments.

During the field enumeration, breast height diameters of all trees in the plot were measured with a caliper, using the mean of two

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[1]) see for example, PETERS, 1997; SCHRÖDER, 1998; OTTO, 2002.

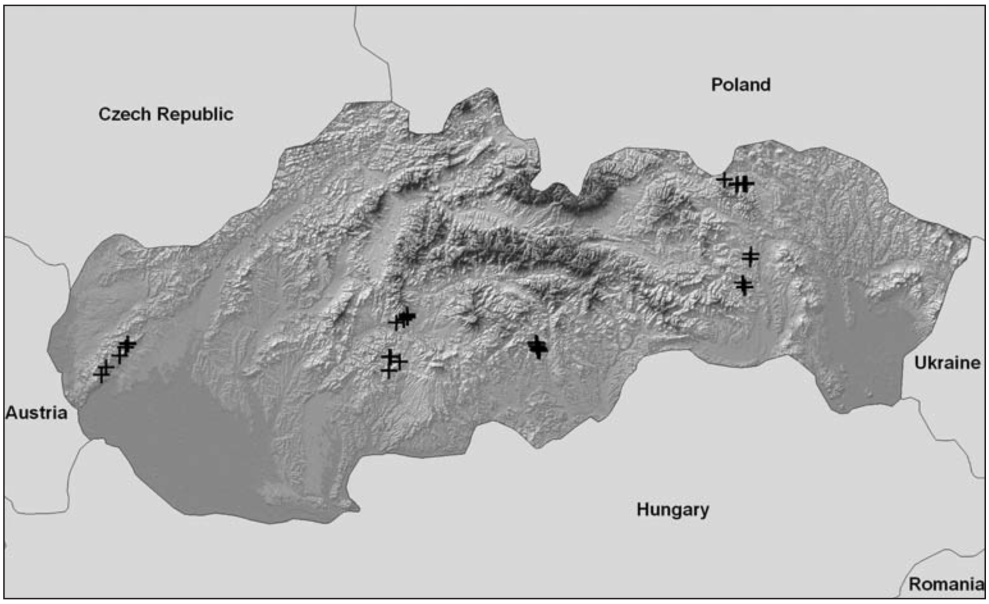

Fig. 1
Map of the beech growth and yield plots in Slovakia.
Lage der Buchenversuchsflächen in der Slovakischen Republik.

Tab. 1
Summary statistics of the dataset used in this study.
Kennwerte des Datensatzes.

Stand variable	Mean	Minimum	Maximum	Standard Deviation
Stand age (t, years)*	107	23	190	40.9
Dom. height (H, m)	29.3	9.9	43.3	7.3
Basal area (G, m^2/ha)	35.03	17.07	52.68	6.97
Site index (H at age 100)	26.3	12.0	32.0	3.9
Number of trees per ha (N)	1298	101	21167	2408
$G_{removed}/G_{total}$ (%)	12.1	0	54.9	8.1
$N_{removed}/N_{total}$ (%)	18.2	0	75.6	13.1

* age from natural regeneration.

measurements, at a height of 1.3 m and the exact place of measurement was marked. Total heights were assessed for selected trees. In addition, descriptive variables such as the relative social position within the plot were assessed. Most of the plots are still being remeasured. The variable $G_{removed}/G_{total}$ (%) indicates the weight of a harvest event. $N_{removed}/N_{total}$ (%) divided by $G_{removed}/G_{total}$ (%) shows the selectivity or "type" of a harvest event.

3. MODELING APPROACH

The analysis in this study includes two different approaches. We first attempt to estimate maximum density using a nonlinear and a linear model. These estimates use quantile regression implemented in the package *quantreg* of *R* version 2.9.1. The second approach involves transition functions to project an observed set of state variables to future states (García, 1988; 1994; 2003). In the present study, the transition functions refer to dominant height, stand basal area and tree survival (natural decline in tree number). The maximum density estimate is considered a useful supplements to the estimates of tree survival.

3.1 Estimating maximum density

One traditional approach to estimating maximum density is to relate number of trees per unit area (N) with average tree size represented by the mean quadratic dbh (Dq), or other tree size variables (like volume or average tree height) and to establish a "limiting relationship" (see, for example, Clutter et al., 1983). For the beech dataset, the nonlinear relationship between the number of trees per ha (N) and the average tree size (Dq) for the 0.975 quantile is $N = 489004.06 \cdot Dq^{-2.026}$. This result is presented in *Fig. 2*. The value of the exponent (–2.026) typically deviates from "Reineke's constant" –1.605. This exponent, derived by fitting a regression to Douglas fir stands has been incorrectly assumed to be valid for a range of forest types (see Gadow, 1987; 2004 for details).

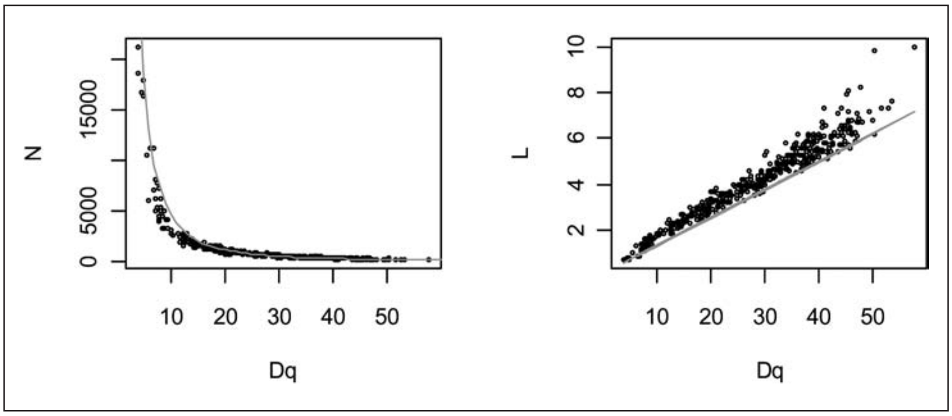

Fig. 2

Maximum density, evaluated by quantile regression, using the traditional approach with tau = 0.975 percentile and a nonlinear fit (left) and Nilsson's method with tau = 0.025 percentile linear fit.

Mit Hilfe der Quantil-Regression geschätzte Maximaldichte.
Links: der herkömmliche Ansatz mit tau = 0.975; rechts: Nilsson's Ansatz mit tau = 0.025.

Tab. 2

The three best transition functions for estimating beech tree survival, dominant height and basal area. N_0, H_0 G_0 and N, H, G are the number of trees per hectare, the dominant heights (m) and the basal areas (m^2/ha) at the current age t_0 and future age t respectively.

Die drei besten Schätzfunktionen für Stammzahl, Oberhöhe und Grundfläche. N_0, H_0 G_0 und N, H, G sind die Stammzahl pro ha, die Oberhöhe (m) und die Grundfläche (m^2/ha) jeweils mit dem Ausgangsalter t_0 und dem zukünftigen Alter t.

Reference	**Equation**	symbol
Clutter & Jones (1980); Woollons (1998)	$N=\left[N_0^{-0.5}+a_1\cdot\left[\left(\frac{t}{100}\right)^2-\left(\frac{t_0}{100}\right)^2\right]\right]^{-2}$	[N1]
Tomé et al. (1997)	$N=N_0\cdot e^{a_1\cdot(t-t_0)}$	[N2]
Zunino and Ferrando (1997)	$N=N_0\cdot\left(\frac{t}{t_0}\right)^{a_1}\cdot e^{a_2\cdot(t-t_0)}$	[N3]
No reference	$H=H_0\cdot\left(\frac{t}{t_0}\right)^{b_1}$	[H1]
Richards (1959) Pienaar & Turnbull (1973)	$H=H_0\cdot\left(\frac{1-e^{b_1\cdot t}}{1-e^{b_1\cdot t_0}}\right)^{b_2}$	[H2]
McDill and Amateis (1992)	$H=\frac{b_1}{1-(1-\frac{b_1}{H_0})\cdot(\frac{t_0}{t})^{b_2}}$	[H3]
No reference	$G=G_0\cdot\left(\frac{t}{t_0}\right)^{c_1}\cdot\left(\frac{N}{N_0}\right)^{c_2}$	[G1]
No reference	$G=N^{c_1}\cdot\left(\frac{H}{H_0}\right)^{c_2}\cdot\left(\frac{G_0}{N_0}\right)^{c_3}\cdot e^{c_4\cdot(t-t_0)}$	[G2]
No reference	$G=c_0\cdot N^{c_1}\cdot\left(\frac{H}{H_0}\right)^{c_2}\cdot\left(\frac{G_0}{N_0}\right)^{c_3}\cdot e^{c_4\cdot[\ln(H)/t-\ln(H_0)/t_0]}$	[G3]

NILSON (2006) suggested to use average spacing between trees (L) which he calculated as $L = \sqrt{10000/N}$ where N is number of trees per ha and L is measured in metres. *L* and *Dq* have the same units, which is convenient, especially when interpreting species-specific size-related density of mixed stands. The linear relationship between average spacing (*L*) and average tree size (*Dq*) for the 0.025 percentile is $L = 0.1332 + 0.1217 \cdot Dq$ for the beech dataset (*Fig. 2*).

Estimates of maximum density are a useful part (some may claim a basic requirement) of any growth simulator. However, it often happens that trees are subject to density-induced mortality before the theoretical maximum density is reached. Therefore, it is useful to complement, or replace, the estimates of maximum density with an estimate of tree survival.

3.2 Transition functions for tree survival, dominant height and basal area

Several authors have pointed out the desirable attributes of transition functions in forest growth research (BAILEY and CLUTTER, 1974; CLUTTER et al., 1983; AMARO et al., 1997; GADOW and HUI, 1999; CIESZEWSKI, 2001). After evaluating a substantial set of potentially useful models, the three best ones were identified for each of the key factors which are used for describing the response of a stand to a harvest event: tree survival, dominant height and basal area. Estimates of these three quantities can be used to calculate other variables, like volume or mean tree size. Stand volume can be estimated from basal area and height, and sometimes including trees per ha as an additional variable (BEEKHUIS et al., 1966). The quadratic mean diameter is calculated directly from basal area and surviving number of trees.

As mentioned before, it is necessary to complement the estimates of maximum density with an estimate of tree survival. Previous studies have shown that tree survival may be affected by current density and age or size. For this reason a number of transition functions, derived from specific differential equations, were evaluated. The three best performers, based on bias and precision criteria, are shown in *Tab. 2*. Readers who are interested in a more throrough analysis of tree survival are referred to WOOLLONS (1998), ÁLVAREZ-GONZÁLEZ et al. (2004) and CASTEDO-DORADO et al. (2007). Dominant height is a key component of a forest stand model. The dominant height models which were selected for final analysis, after scrutinizing a range of others, are also shown in *Tab. 2*. Site quality did not have an affect tree survival, but height growth significantly affected basal area growth. The three basal area models, listed in *Tab. 2*, were selected as the best ones from nine different linear and nonlinear equations. In this analysis, any direct effect of thinning on stand basal area growth is disregarded.

3.3 Model Comparison and Evaluation

This section identifies the final set of one equation each for estimating tree survival, dominant height growth and basal area growth. This basic set will entail the most essential information for silvicultural planning and economic analysis.

The comparison of the estimates for the different equations fitted for each stand variable was based on numerical and graphical analyses of the residuals. Three statistical criteria were examined: bias ($\bar{E}$), which tests the systematic deviation of the model from the observations; root mean square error (*RMSE*), which gives the pre-

Tab. 3

The coefficients and evaluation statistics of the nine equations listed in *Tab. 2*.

Koeffizienten und Beurteilungskriterien für die neun Funktionen in *Tab. 2*.

Model	Parameter	Estimate	Ē	RMSE	MEF	AIC	BIC
N1	a_1	0.0031^{***}	13.1431	77.5648	0.9999	1757.09	1763.14
N2	a_1	-0.0040^{***}	16.0137	80.4722	0.9988	1768.28	1774.33
N3	a_1	-0.2322^{***}	0.5907	74.015	0.999	1743.84	1752.91
	a_2	$0.0024^{\dagger}$					
H1	b_1	0.8946^{***}	0.0028	1.0586	0.9772	451.7	457.7
H2	b_1	-0.00972^{***}	0.0874	0.9281	0.9825	412.6	421.7
	b_2	1.4084^{***}					
H3	b_1	41.2167^{***}	0.1032	1.5908	0.9485	576.48	585.55
	b_2	-0.2012^{***}					
G1	c_1	1.711^{***}	0.178	1.437	0.946	545.56	554.63
	c_2	0.7161^{***}					
G2	c_1	0.9736^{***}	0.0717	1.6033	0.9331	580.81	595.93
	c_2	0.6396^{***}					
	c_3	0.9391^{***}					
	c_4	0.01139^{***}					
G3	c_0	1.6349^{***}	0.0051	1.5017	0.9413	561.88	580.02
	c_1	0.8914^{***}					
	c_2	0.6300^{***}					
	c_3	0.9178^{***}					
	c_4	-26.7108^{***}					

Significant codes: *** $p < 0.001$, ** $p < 0.01$, * $p < 0.05$, † $p < 0.06$. **AIC = AKAIKE's information criterion**, published by AKAIKE (1974) under the name of "an information criterion" (**AIC**) is a measure of the goodness of fit of an estimated statistical model; it describes the tradeoff between bias and variance in model construction. BIC = the **Bayesian information criterion or Schwarz Criterion** (also known as **SBC**, **SBIC**) is a criterion for model selection among a class of parametric models with different numbers of parameters.

cision of the estimate and model efficiency (*MEF*), which shows the proportion of the total variance that is explained by the model, adjusted for the number of model parameters and the number of observations. The expressions of these statistics are as follows:

Bias	Precision	Model Efficiency
$\overline{E} = \frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)}{n}$	$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{n-p}}$	$MEF = 1 - \frac{(n-1)\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{(n-p)\sum_{i=1}^{n}(y_i - \bar{y}_i)^2}$

where, y_i, and $\hat{y}_i$ are $\bar{y}_i$ the observed, predicted and average values of the dependent variable, respectively; n is the total number of observations used in fitting the function; and p is the number of model parameters. The coefficients and evaluation statistics of the nine models are presented in *Tab. 3*.

With one exception, all the coefficients are highly significant. Among the nine individually fitted models, equations N3, H2 and G1 showed the best performance. N3 was selected although the parameter a2 was only significant at the level p = 0.053. Neglecting the exponential term $e^{b2 \cdot (t - t0)}$ of this equation resulted in a poorer fit. Therefore, it was decided to maintain the original formulation by ZUNINO and FERRANDO (1997). Equation H2 is the best choice for estimating dominant height growth, closely followed, however, by the much simpler model H1. In the graphical analysis N3, H2 and G1 also showed the best performance.

3.4 Systemfit for stand dominant height, number of trees and basal area

The transition functions for dominant height, basal area and natural decline of number of trees together define a system of three equations. Normally, one would try to solve such a system of equations independently, e.g. using the least squares method for each equation separately. But in *Seemingly Unrelated Regression* (SUR) models, which represent a special case of the generalized least squares approach, it is assumed that the error terms from different equations are correlated. According to the general least squares theory, which takes covariances of errors into account, such systems should be solved simultaneously as a whole set of equations to minimize the total sum of square errors in the whole system. In this study, the analysis was performed using the *systemfit* package (HENNINGSEN and HAMANN, 2007) of the *R* statistical software. The results of the parameter estimates for the three equations fitted separately and simultaneously are shown in *Tab. 4*.

The variable *N* appears both on the left [N3] as a prediction and right hand side [G1] as an observation. As expected, the parameter values resulting from the simultaneous SUR estimation differ from those of the separate estimation. However, the differences are very small. Thus, there are two main motivations for using *SUR*. The first one is to gain efficiency in estimation by combining information on different equations. The second motivation is to impose restrictions that involve parameters in the different equations. SRIVASTAVA and GILES (1987) present a thorough treatment of the theory.

The graphical analysis of the residuals for the *SUR* method is presented in *Fig. 3*. The distributions of the residuals appear to be random around zero. There are no obvious systematic discrepancies. The plots of predicted versus observed values show a good fit for each of the three equations.

The evaluation shows that the beech model is comparatively robust. The accuracy is rather good, which is surprising in view of the fact that the data include the response to extreme harvest events.

4. APPLICATION EXAMPLES

An important objective of a growth model is to estimate the development of a forest between successive harvest events. In this section, we present a few examples of the practical application of the model (*Tab. 5*). The examples illustrate applications involving different stand ages and silvicultural treatments. The model can be easily implemented in a spreadsheet.

Each harvest event is characterized by the thinning weight (how much? i.e. the proportion of the basal area removed) and the thinning type (which trees? i.e. the proportion of the stems per ha removed divided by the proportion of the basal area removed). Each set of initial heights, trees and basal areas per ha are taken from one of the available experimental plots. The shortest and longest prediction intervals are 5 and 9 years respectively.

Tab. 4

Parameter estimates for the three equations fitted separately and simultaneously using SUR.

Parameterwerte für die drei Funktionen separat (links) und simultan (rechts) geschätzt.

Model	Separate OLS*	Simultaneous SUR**	
Tree survival	$N = N_0 \cdot \left(\frac{t}{t_0}\right)^{-0.2322} \cdot e^{0.0024 \cdot (t - t_0)}$	$N = N_0 \cdot \left(\frac{t}{t_0}\right)^{-0.2140} \cdot e^{0.0020 \cdot (t - t_0)}$	[N3]
Dominant height	$H = H_0 \cdot \left(\frac{1 - e^{-0.00972 \cdot t}}{1 - e^{-0.00972 \cdot t_0}}\right)^{1.4084}$	$H = H_0 \cdot \left(\frac{1 - e^{-0.00929 \cdot t}}{1 - e^{-0.00929 \cdot t_0}}\right)^{1.3653}$	[H2]
Basal area	$G = G_0 \cdot \left(\frac{t}{t_0}\right)^{1.711} \cdot \left(\frac{N}{N_0}\right)^{0.7161}$	$G = G_0 \cdot \left(\frac{t}{t_0}\right)^{1.7322} \cdot \left(\frac{N}{N_0}\right)^{0.8669}$	[G1]

* OLS = Ordinary Least Squares (The OLS estimates give the best linear unbiased estimate if the Gauss-Markov assumptions hold for all the equations); ** SUR = Seemingly Unrelated Regression (an extension of the linear regression model which can be used for analyzing a system of multiple equations with cross-equation parameter restrictions and correlated error terms, first published by ZELLNER, 1962).

The first two lines in *Tab. 5* show the observed initial and final values of N, G and H, following the harvest event at the initial age. The harvest event is defined by the remaining N and G. The "initial" line is repeated three times, once for each of three of three alternative harvest events which are specified in the "Remaining" column. The model estimates following the three different harvest events are listed in the "predicted" line and the "Total" column.

Each of the six alternatives A can be compared directly with the observed estimates because the harvest events are the same as in the experiments. These estimates are very close, except for the basal area predictions in the very young experiments (P102; VP72). Observations for alternatives B and C are not available and the estimates cannot be compared. The predictions for B and C merely illustrate the use of the model for a wide range of stand conditions and silvicultural treatments.

5. DISCUSSION AND CONCLUSIONS

The objective of this paper was to develop a dynamic stand growth model for Beech (*Fagus sylvatica*) forests which can be used for analysing alternative silvicultural options for varying prediction intervals. Such a tool is an essential basis for implementing new paradigms in forest design, such as the multiple path concept (GADOW et al., 2009). The two important requirements of the model are accurate predictions and ease of specifying silvicultural options. Based on the good fit of the model to the empirical observations, the first requirement has been met, at least within the range of the silvicultural treatments practiced in the field plots. The second requirement is also satisfied, because a harvest event can be specified in terms of the thinning weight (the proportion of the basal area removed) and in terms of the thinning type (the proportion of the stems per ha removed divided by the proportion of the basal area removed). This rather simple method of specifying a harvest event may be sufficiently accurate for most applications.

Tab. 5

Stand growth after three different harvest events in six experimental plots (TVP5; VP11; VP34; VP72; P102; P160). The characteristics of each harvest event are shown in the column "Remaining", indicating the basal area and stems per ha remaining after each tinning.

Beobachtete und geschätzte Reaktion auf unterschiedliche Eingriffstärken und -arten in 6 Versuchsflächen (TVP5; VP11; VP34; VP72; P102; P160). Die Eingriffstärke und -art ergibt sich aus der verbleibenden Stammzahl und Grundfläche in der Spalte „Remaining".

TVP5/4/1981-97			N	Total G	H	Remaining N	G
initial		80	778	33.0	28.6	659	30.7
observed		96	658	41.8	34.3		
initial	A	80	778	33.0	28.6	659	30.7
predicted		96	654	42.1	33.5		
initial	B	80	778	33.0	28.6	622	28.1
predicted		96	773	45.0	33.5		
initial	C	80	778	33.0	28.6	700	28.1
predicted		96	773	45.0	33.5		
VP11/2/1971-76			N	Total G	H	Remaining N	G
Initial		65	1564	40.4	23.0	1264	32.9
Observed		70	1264	36.7	24.0		
Initial	A	65	1564	40.4	23.0	1264	32.9
Predicted		70	1257	37.2	24.7		
Initial	B	65	1564	40.4	23.0	1251	34.4
Predicted		70	1257	39.2	24.7		
Initial	C	65	1564	40.4	23.0	1408	34.4
Predicted		70	1244	35.1	24.7		
VP34/3/1976-81			N	total G	H	Remaining N	G
Initial		148	914	28.4	20.3	745	23.3
Observed		153	745	25.1	21.7		
Initial	A	148	914	28.4	20.3	745	23.3
Predicted		153	747	24.8	20.7		
Initial	B	148	914	28.4	20.3	731	24.2
Predicted		153	747	26.1	20.7		
Initial	C	148	914	28.4	20.3	823	24.2
Predicted		153	733	23.2	20.7		
VP72/1/1968-73			N	total G	H	Remaining N	G
Initial		23	21167	26.1	10.0	18518	21.0
Observed		28	17861	32.3	11.5		
Initial	A	23	21167	26.1	10.0	18518	21.0
Predicted		28	17933	28.7	12.7		
Initial	B	23	21167	26.1	10.0	16934	22.2
Predicted		28	17933	32.8	12.7		
Initial	C	23	21167	26.1	10.0	19050	22.2
Predicted		28	16399	27.4	12.7		
P102/3/1979-88			N	total G	H	remaining N	G
Initial		47	7706	34.0	18.1	2637	22.8
Observed		56	2631	34.3	24.4		
Initial	A	47	7706	34.0	18.1	2637	22.8
Predicted		56	2586	30.4	21.9		
Initial	B	47	7706	34.0	18.1	6165	28.9
Predicted		56	2586	18.4	21.9		
Initial	C	47	7706	34.0	18.1	6935	28.9
Predicted		56	6046	34.7	21.9		
P160/2/1978-87			N	total G	H	remaining N	G
Initial		169	316	33.8	31.5	302	32.9
Observed		178	302	36.0	32.7		
Initial	A	169	316	33.8	31.5	302	32.9
Predicted		178	304	36.2	32.4		
Initial	B	169	316	33.8	31.5	253	28.7
Predicted		178	304	36.9	32.4		
Initial	C	169	316	33.8	31.5	284	28.7
Predicted		178	255	28.5	32.4		

A frequently used approach for multivariate growth modelling problems is Mixed Model Theory. With mixed models one can analyse individual deviations from the population average with the help of fixed and random effects. It is also possible to model intercorrelations between different growth components at different hierarchical levels (HALL and CLUTTER, 2004). Mixed models provide a flexible framework, including methods and algorithms for the analysis and prediction of tree growth. We did not use this method in the present study because the experimental plots are not randomly distributed over the potential application area. Furthermore, individual tree data were not available and the analysis would have to be limited to a single level. Finally, the error structure aspects addressed by mixed modelling are trivial compared with those arising from the compounding of environmental disturbances over time, plus measurement errors. A proper error structure would involve hierarchical stochastic differential equation based modelling (see, for example, SEBER and WILDE, 2003). However, as suggested by the SUR results, we suspect that with an adequate (deterministic) model specification and suitable data, the estimation procedure makes little difference. No amount of statistical wizardry can compensate for a misspecified model.

The comparisons in *Tab. 5* show very good estimates, except for the basal area predictions in the very young stands (P102; VP72). Observations for alternatives B and C are not available and the estimates are merely used to illustrate the practical application, e.g. for economic analysis. Given the good model fit and the wide range of stand conditions and silvicultural treatments in the 47 experiments, it can be assumed that those predictions are also reasonable. Future evaluations with extremes of density, site quality and silvicultural treatment are recommended. The longest measurement interval in the observation dataset is 19 years, but the most frequent interval was 5 years. It is assumed that possible thinning effects may be disregarded if prediction intervals do not exceed 5 years, which is the normal interval between successive harvest events.

The model developed in this study represents a compromise between the inflexible but trusted yield tables and the more sophisticated individual tree models. The quadratic mean diameter, which is required for estimating log yields, can be calculated directly from basal area and number of trees harvested. Total volume is usually estimated from basal area and dominant tree height. Thus, the set of three equations, which is easily implemented in a spreadsheet, provides the minimum and at the same time sufficient information for silvicultural planning and economic analysis of alternative treatments. The stand-level growth predictions, together with the maximum density estimates, provide a solid base for individual tree growth models. This topic will be the subject of a future study involving a hierarchical system of beech models which, based on a more complete Central European dataset, will represent a logical continuation of the work by SCHWAPPACH (1911), WIEDEMANN (1931) and SCHOBER (1972).

6. ABSTRACT

The change from the traditional yield tables to individual tree growth models for beech has been met with enthusiasm as well as scepticism. This paper presents a dynamic stand growth model for Beech (*Fagus sylvatica*) forests, representing a compromise between the inflexible yield tables and the less robust individual tree models. The study is based on the complete beech dataset provided by the Slovak Forest Research Institute in Zvolen, which includes 47 long-term research plots. The experiments were remeasured between one and six times, providing up to five interval measurements per plot. Individual tree data are not available at this stage. The first step involved estimating the maximum stand density. Then the initial stand conditions at any point in time were defined by the three state variables dominant height (H), basal area (G) and number of trees per hectare (N). Transition functions are used to project the state variables at any particular time. A large number of different functions were first fitted to the data and evaluated. Among these, the nine best ones (three each for H, G and N) were compared using numerical and graphical methods. The parameters of the three best ones among these (one each for H, G and N) were first estimated independently, and then simultaneously using the seemingly unrelated regression approach. Following the analysis, several application examples are presented which demonstrate the practical use in silvicultural planning and forest design.

7. Zusammenfassung

Titel des Beitrages: *Schätzung von Wachstum und Maximaldichte in slovakischen Buchen-Dauerversuchsflächen.*

Dieser Beitrag beschreibt die Entwicklung eines Wachstumsmodells für die Schätzung der Reaktion von Buchenbeständen auf unterschiedliche forstliche Eingriffe. Der Datenbestand der Slovakischen Forstlichen Versuchsanstalt in Zvolen mit 47 langfristigen Versuchsflächen auf unterschiedlichen Standorten, mit jeweils bis zu 6 Wiederholaufnahmen, stand zur Verfügung. Das Modell umfasst Schätzfunktionen für die Maximaldichte und natürliche Mortalität, sowie Grundflächen- und Höhenzuwachs der Bestände, und stellt einen Kompromiss zwischen den Ertragstafeln einerseits und den Einzelbaummodellen andererseits dar. Die Schätzfunktionen prognostizieren die Entwicklung eines Initialzustandes in Reaktion auf bestimmte Eingriffe. In einem ersten Schritt wurden zahlreiche aus der Literatur bekannte Modelle untersucht. Die neun besten Schätzfunktionen (jeweils drei für Oberhöhe, Grundfläche und Stammzahl) wurden dann mit Hilfe numerischer und graphischer Methoden miteinander verglichen. Die Parameter der drei besten Funktionen (jeweils eine für Oberhöhe, Grundfläche und Stammzahl) wurden zunächst unabhängig und schließlich simultan geschätzt, unter Verwendung der SUR (seemingly unrelated regression) Methode. Abschließend wird die praktische Anwendung der Schätzfunktionen mit Hilfe einiger Beispiele demonstriert.

8. Résumé

Titre de l'article: *Estimation de la croissance du hêtre et de sa survie. Etude fondée sur des expérimentations de longue durée en Slovaquie.*

Cette contribution décrit le développement d'un modèle de croissance pour estimer la réaction de peuplements à différentes interventions forestières. On avait à notre disposition la base de données de l'Institut de Recherche Forestière de Zvolen comportant 47 places d'expérience de longue durée sur diverses stations, avec selon les cas jusqu'à 6 répétitions. Le modèle comprend des fonctions d'estimation de la densité maximale du peuplement et la mortalité naturelle ainsi que la croissance en surface terrière et en hauteur des peuplements et représente un compromis entre les tables de production d'une part et les modèles d'arbre isolé d'autre part. Les fonctions d'estimation pronostiquent le développement d'un peuplement initial en réaction à certaines interventions. Dans un premier temps on a cherché de nombreux modèles dans la littérature. Les neuf meilleures fonctions d'estimation (chaque fois trois pour la hauteur dominante, la surface terrière et le nombre de tiges à l'hectare) furent comparées les unes aux autres à l'aide de méthodes numériques et graphiques. Les paramètres des trois meilleures fonctions (chaque fois une pour la hauteur dominante, la surface terrière et le nombre de tiges à l'hectare) furent ensuite estimés d'abord indépendamment et enfin simultanément, en utilisant la mèthode SUR (seemingly unrelated regression). En conclusion on présente l'utilisation pratique des fonctions d'estimation à l'aide de quelques exemples. R.K.

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